The Value of Delayed Gratification

The rate of pure time preference or the discount rate for future gratification plays a critical role in analyzing long-horizon economic analyses. Greenhouse gas (GHG) abatement policy modeling is an example. Studies that assume the rate of pure time preference, \( \rho \), is zero found that a 40% reduction in GHG emissions optimal. In contrast, studies assuming \( \rho = 3\% \) found that only modest emission abatement can be justified economically. More recently, Nicholas Stern’s 2007 book, The Economics of Climate Change: The Stern Review, essentially assumes \( \rho = 0 \), but its rejection of discounting future utility has drawn criticism.

There exist two schools regarding how the value of \( \rho \) should be determined. Some economists believe that \( \rho \) should be zero on moral grounds. Other economists argue that the value of \( \rho \) must be consistent with revealed intertemporal choices. Specifically, the latter school believes that \( \rho \) should be determined according to the following equilibrium condition for the Ramsey growth model (the Deterministic Ramsey Rule):

\[
1 + r = (1 + \rho)(1 + g)^\alpha
\]

where \( r \) is the rate of return on capital, \( \alpha \) is (the absolute value of) the elasticity of the marginal utility with respect to consumption, and \( g \) is the consumption growth rate. Presumably, the value of \( \rho \) can be readily inferred from the Deterministic Ramsey Rule if we know the values of \( r \) and \( g \) (which are observable in principle) and have a reasonable estimate of \( \alpha \).

The value of \( g \) is relatively well understood. For example, in Rajnish Mehra and Edward Prescott’s 1985 article, “The Equity Premium Puzzle,” in the Journal of Monetary Economics, the estimated average annual growth rate of real per-capita consumption is 1.8% based on the annual U.S. data from 1889 to 1978. However, determining the values of \( r \) and \( \alpha \) is less straightforward.

First, a question arises as to which \( r \) should be put in the left-hand side of the Ramsey Rule because of the coexistence of multiple asset returns. Different values of \( r \) would generate different values of \( \rho \) given \( \alpha \) and \( g \). For example, suppose \( \alpha = 1 \) (corresponding to the logarithmic instantaneous utility function) and \( g = 1.8\% \). Then an \( r \) of 7.0% (corresponding to the mean equity return) implies \( \rho = 5\% \), but \( r = 0.8\% \) (corresponding to the risk-free return) implies \( \rho = -1\% \).

Second, estimates of \( \alpha \) are very diverse, and its appropriate value depends on the context in which it is applied. Specifically, the reciprocal of \( \alpha \), namely, the elasticity of intertemporal substitution, is the parameter of focus in the macro literature on life-cycle consumption. Letting \( \alpha \) be one (i.e., the logarithmic utility) is the preferred choice in macroeconomic models, which seems to be due to the evidence of relatively strong intertemporal substitution. On the other hand, \( \alpha \) itself, as the coefficient of relative risk aversion, is the parameter of focus in the finance literature on asset returns and risks. Data in this context imply a value of \( \alpha \) much larger than one.

These conceptual difficulties inherent in using the Deterministic Ramsey Rule to infer \( \rho \) suggest that one should incorporate uncertainty and multiple asset returns into the analysis and separate the two preference parameters traditionally represented by a single parameter. This is exactly what is done by PERC Research Scientist Liqun Liu in PERC Working Paper #1108.

When the gross consumption growth rate \( 1 + \tilde{g} \) follows a lognormal distribution, the Generalized Uncertainty Ramsey Rule becomes

\[
\ln(1 + i_B) = \ln(1 + \rho) + \alpha_u \mu + \frac{1}{2} \sigma^2 (\alpha_u - \alpha_u - \alpha_u \alpha_u) ,
\]

where \( i_B \) is the risk-free bond return, \( \mu \) and \( \sigma^2 \) are the mean and variance of the normal distribution \( \ln(1 + \tilde{g}) \),
respective, and $\alpha_u$ and $\alpha_v$ are the reciprocal of the elasticity of intertemporal substitution and the coefficient of relative risk aversion, respectively.

To infer $\rho$ from the Generalized Uncertainty Ramsey Rule under various combinations of $\alpha_u$ and $\alpha_v$, Liu uses data on asset returns and consumption growth from the U.S. economy for the 1889-1978 period. Based on these data, $i_b = 0.8\%$, $\mu = 0.0172$ and $\sigma = 0.00125$.

Liu’s main numerical findings are: (i) for $\alpha_u$ (the reciprocal of the elasticity of intertemporal substitution) equal to or less than one, the value of $\rho$ lies within ±1% of zero regardless of the relative risk aversion within a plausible range; and (ii) for larger $\alpha_u$, the value of $\rho$ tends to lie in the negative zone. These results contradict the widely-held belief in the environmental economics literature that the inferred $\rho$ must be significantly larger than zero and suggest that it is appropriate to use $\rho = 0$ as a benchmark for long-term economic analysis.

These findings are initially obtained with specific parameter values on consumption growth and the risk-free return and with the assumption that the gross consumption growth rate $1 + \tilde{g}$ follows the lognormal distribution. Liu then investigates how sensitive these findings are to the relevant parameter values and to the assumption of lognormal distribution. He finds that those findings remain the same.

Perception of Mortality and Bequests

Over the next 25 years, as the population of the aged swells, the incidence of mortality-related asset transfers is expected to increase substantially. Additionally, as the retirees acclimate to this stage in their life, their consumption patterns will deviate from that of their working years’. What are the factors that affect those consumption decisions? How will those decisions affect the economy and the financial status of their inheritors?

In PERC Working Paper #1102, PERC Research Fellow Li Gan, along with coauthors Guan Gong, Michael Hurd, and Daniel McFadden investigate how retirees’ expectations about their longevity inform their consumption decisions. They apply a standard dynamic life-cycle model to individual survey data and find that bequest motives are relatively unimportant when formulating consumption decisions. This result suggests that bequests are generally not predetermined but accidental.

Previous research on this topic relied on life tables (life expectancy figures based solely on age) to determine the retirees’ expectations. Retiree heterogeneity in health or genetics, however, could invalidate those results if retirees’ differences resulted in different longevity expectations. Data suggest that, indeed, using individual expectations generates results more consistent with actual outcomes.

Though research generally indicates that bequests play a significant role in wealth accumulation, investigations into the bequest motive, whether retirees leave bequests deliberately or as a result of necessarily imperfect planning, persist. The authors use the existence of children to identify the bequest motive—believing that retirees with children will have less inclination to consume all of their wealth before they die.

The paper uses the Asset and Health Dynamics among Oldest Old (AHEAD) dataset; it includes people born between 1890 and 1923 and their spouses. Most importantly, it includes respondents’ expectations of living to certain ages. Earlier work found that these responses generally conform to the life tables but also account for relevant risk factors.

The authors adopt and modify a standard life-cycle model with bequests. It is the sum of expected utility from consumption and
expected utility derived from making a bequest for each period. They include a borrowing constraint to disallow borrowing against Social Security benefits. They further model the marginal utility from a bequest as a function of the number of children the retiree has. To estimate, they take two responses from each individual (corresponding to the 2nd and 3rd waves of the survey); they apply their model to the responses from the 2nd wave to predict the wealth in the 3rd wave. They find the parameter values that minimize the difference between predicted 3rd wave wealth and actual 3rd wave wealth.

They next describe the three possible types of retirees: those that leave bequests, those that exhaust their wealth just as they die, and those that exhaust their wealth prior to their death and survive on annual income alone. They use the respondents’ subjective survival curves to calculate the critical level of wealth (the point at which any greater level of wealth will result in a positive bequest). They then use this value to calculate the trajectories of both consumption and wealth. Two approaches are available for minimizing the difference between the predicted final wealth and actual final wealth—a mean square loss function and an absolute value loss function. The latter has the advantage of de-emphasizing outliers.

Gan et al. restrict their sample to those who are alive and single in waves 2 and 3 and have positive total wealth in both waves. The final sample size is either 1,903 or 1,752 depending on whether the authors use total wealth or non-housing wealth. The sample comprises those who were 70 years old or older in 1993; the subsequent waves were conducted in 1995, 1998, and 2000.

The survey asked respondents what they believed the probability of surviving to a target age 10-15 years into the future. Though the responses were generally in line with life tables, many respondents entered the unrealistic 0 or 100. To account for this, the authors employ an optimism index. The optimism index assigns the respondents a value based on how their response compares to the life tables. The index is then converted into a subjective survival probability.

For the model, the authors excluded housing wealth. Though some may argue that the aged consider the value of housing when planning for their death, the authors found that excluding it didn’t markedly affect their results, and they believed that since housing wealth is not readily consumable, it wouldn’t fit in their model.

They find that using the life tables instead of subjective mortality suggests that retirees prefer consuming in the future relative to current consumption. Since respondents generally believe they’ll live longer than the life tables predict, though, this result was expected. Because they’re expecting to live longer, they save more/consume less, which gives the appearance of a lower discount rate. They also find that using the mean square loss function results in high bequest motives while the alternative results in almost zero bequest motives. The authors attribute this to very high wealth outliers driving the former’s results.

Finally, the authors compared the outcomes their model would predict to actual outcomes. Since they used waves 2 and 3 of the survey to estimate the parameters, they retained the 4th wave to conduct this assessment of their results. They compare estimates based on using life table data versus subjective survival probabilities and on the two methods of estimation. Generally they find that the absolute value loss function and subjective survival probabilities come closer to matching the observed values.

As the baby boom generation transitions into their retirement years, the inclination to leave wealth to their heirs will affect, even determine, their consumption decisions. To better understand the impact to the economy, we must understand how predisposed they are to passing on their wealth and how accurately they can predict their mortality. This paper uses the existence of children to identify retirees’ motives and finds that for most, the desire to leave a bequest is weak. Furthermore, the authors’ results indicate that when studying end of life decisions, assuming expectations conform to life table data may produce erroneous results.